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**2835 [1920, 273]. Proposed by J. L. RILEY, Stephenville, Texas.**

If $x$, $y$, $z$, $u$ are finite, and not all zero, and satisfy the equations

$$x = by + cz + du, \qquad y = ax + cz + du, \qquad z = ax + by + du, \qquad u = ax + by + cz,$$

and if none of the quantities $a$, $b$, $c$, $d$ have the value $-1$, then will

$$\frac{a}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} + \frac{d}{1+d} = 1.$$

## SOLUTION BY H. L. OLSON, University of Michigan.

The necessary and sufficient condition that these equations shall have a set of solutions not all zero is that the determinant of the coefficients,

$$\begin{vmatrix} -1, & b, & c, & d \\ a, & -1, & c, & d \\ a, & b, & -1, & d \\ a, & b, & c, & -1 \end{vmatrix} = 0.$$

If we expand this determinant and divide by $(a + 1)(b + 1)(c + 1)(d + 1)$, which is not zero by hypothesis, we find that the condition can be written

$$\frac{a}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} + \frac{d}{1+d} = 1.$$

Also solved by P. J. DA CUNHA, ARTHUR PELLETIER, A. V. RICHARDSON, C. H. RICHARDSON, H. S. UHLER, and C. C. WYLIE.

**2857 [1920, 377]. Proposed by the late L. G. WELD.**

A savings bank offers to pay 3% interest on deposits, the said interest to be continuously compounded, *i.e.*, compounded at infinitesimal intervals of time. What would be the amount of $1.00 for one year?

## SOLUTION BY E. J. OGLESBY, Washington Square College, New York University.

The amount of one dollar at the percentage, denoted by the fraction $r$, compounded $n$ times per year is $(1 + r/n)^n = [(1 + r/n)^{n/r}]^r$. Hence

$$A = \underset{n \doteq \infty}{\text{limit}} \left[\left(1 + \frac{r}{n}\right)^{n/r}\right]^r = e^r = e^{.03} = 1.03045.$$

Also solved by H. C. BRADLEY, H. N. CARLETON, ELMER LATSHAW, ARTHUR PELLETIER, W. T. STONE, and A. L. WECHSLER.

**2858 [1920, 428]. Proposed by C. P. SOUSLEY, Pennsylvania State College.**

A boy can split wood as fast as his father can saw, and the father can split twice as fast as the son can saw. How should the money received for their labor be divided?

## SOLUTION[1] BY C. A. BARNHART, University of New Mexico.

Let $x$ = the number of cords of wood that the boy saws in 1 hour, and $kx$ = the number of cords of wood that the boy splits in 1 hour. Then, $2x$ = the number of cords of wood that the father splits in 1 hour, and $kx$ = the number of cords of wood that the father saws in 1 hour. Then, $\frac{1}{x} + \frac{1}{kx} = \frac{k+1}{kx}$ = the number of hours in which the boy will saw and split 1 cord of wood, and $\frac{1}{2x} + \frac{1}{kx} = \frac{k+2}{2kx}$ = the number of hours in which the father will saw and split 1 cord of wood.

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[1] The question is rather vague, but probably this solution makes the most natural interpretation of it: to use the time it takes each to split and saw a cord of wood as a basis for comparison of their work.—EDITORS.

Assuming that for each dollar received by the son, the father will receive $z$ dollars, and noting that the money received will vary inversely as the number of hours required to saw and split 1 cord of wood, we have

$$z = \frac{k+1}{kx} \Big/ \frac{k+2}{2kx} = 1 + \frac{k}{k+2}.$$

From which it is evident that the money received by the father depends directly upon $k$. Since $\lim\limits_{k \doteq 0} z = 1$ and $\lim\limits_{k \doteq \infty} z = 2$, then for $k > 0$, we have $1 < z < 2$.

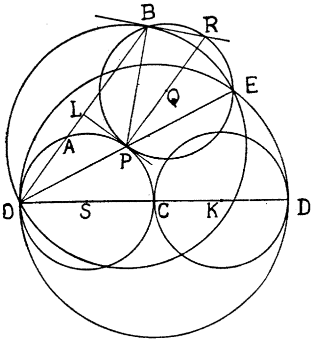
Also solved by C. E. Bardsley, H. C. Bradley, H. N. Carleton, Michael Goldberg, and Daniel Kreth.

**2861 [1920, 428]. Proposed by B. F. FINKEL, Drury College.**

Obtain by plane geometry, *i.e.*, without the use of calculus, a construction for finding points on the envelope of a system of circles whose diameters are chords of a fixed circle passing through a given point on it. Also determine geometrically the nature of the locus.

## I. Solution by Augustus Bogard, College of St. Theresa, Winona, Minn.

The problem of finding the envelope of a system of curves is that of finding a curve that is at every point tangent to some curve of the system. That is, at any point on the envelope the envelope and some curve of the system have a common tangent, and they lie on the same side of it.



Let $O$ be the fixed point, $C$ the center of the given circle, and $OCD$ the diameter from $O$, and let $OE$ be any other chord from $O$, making with $OD$ the angle $\theta$. $OE$ is then a diameter and $P$, its mid-point, is the center of one of the circles of the given system. At $P$, on the further side of $OE$, construct an angle $EPB = 2\theta$, the side of the angle cutting the circle $OE$ at $B$. Then $B$ is a point of the envelope required.

Proof. Draw the circles having $OC$ and $CD$ as diameters, and the circle $PEB$. Let $Q$ be the center and $PQR$ the diameter from $P$ of $PEB$. $PQR$ bisects the angle $EPB$ and this circle is tangent at $P$ to the circle $OC$. Further, chord $PB =$ chord $PO$ and so diameter $PR =$ diameter $OC =$ diameter $CD$. Hence the circle whose center is at $Q$ may be thought of as rolling without slipping on the circle $OC$, and $P$ is the center of the curvature at $B$ of the curve described by this point. It follows that the line $BR$ is tangent to the curve at $B$. It is also tangent at $B$ to the circle $OE$, the circle of the given system, passing through $B$. Hence $B$ is a point on the envelope. This analysis shows that the curve is described in the same manner as the cardioid, and is therefore a cardioid. Its polar equation is easily derived as follows: Considering $O$ as the pole, $OD$ as the initial line, the angle $DOB = \phi$ as the vectorial angle of $B$, and $OB$ its radius vector, we have $OB = OE \cos\theta = OD \cos^2\theta = a(\cos 2\theta + 1) = a(1 + \cos\phi)$, where $a = OC$. That is, $\rho = a(1 + \cos\phi)$, which is the equation of a cardioid.

## II. Solution by Otto Dunkel, Washington University.

Using the notation in A. Bogard's solution, the middle point $P$ of the chord $OE$ of the fixed circle, of radius $OC$, lies on the circle $OC$. It may be shown easily that the tangent at $P$ to the circle $OC$ is parallel to the tangent to the given circle at $E$. Hence the construction given for the point $B$ in the Monthly (*1921*, 182) is equivalent to the following: Draw the tangent at $E$ to the given fixed circle, then the point $B$ is the foot of the perpendicular from $O$ to the tangent. Let $A$ be the point in which $OB$ cuts the circle $OC$, then $AB = CE = a$. Hence the point $B$ is found by the usual method for constructing a cardioid, *i.e.*, by drawing chords $OA$ to the circle $OC$ and prolonging each a length $AB = OC = a$.